Solutions

① $\langle 3,-2,2\rangle \cdot \langle 1,0,1\rangle$

$= 3\cdot 1 + (-2)\cdot 0 + 2\cdot 1$

$= 5$

② $(i+j)\cdot(2j+k) = i\cdot 2j + i\cdot k + j\cdot 2j + j\cdot k$

$= 2(\cancel{i\cdot j})^{0} + \cancel{i\cdot k}^{0} + 2(j\cdot j) + \cancel{j\cdot k}^{0} = 2\|j\|^2 = 2$

$i\cdot j = 0,\ j\cdot k = 0,\ i\cdot k = 0$

③ a) $\langle 3,1,1\rangle \cdot \langle 2,-4,2\rangle = 3\cdot 2 - 4 + 2 = 4$

$\|\langle 3,1,1\rangle\| = \sqrt{9+1+1} = \sqrt{11}$

$\|\langle 2,-4,2\rangle\| = \sqrt{4+16+4} = \sqrt{24}$

$\cos\theta = \dfrac{0}{\sqrt{11}\sqrt{24}} = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

b) $\langle 0,1,1\rangle \cdot \langle 1,-1,0\rangle = 0 - 1 + 0 = -1$

$\|\langle 0,1,1\rangle\| = \sqrt{1+1} = \sqrt{2}$

$\|\langle 1,-1,0\rangle\| = \sqrt{1+1} = \sqrt{2}$

$\cos\theta = \dfrac{-1}{2} \Rightarrow \theta = \dfrac{2\pi}{3}$

④ $\langle 4,-2,7\rangle \cdot \langle b^2, b, 0\rangle = 4b^2 - 2b = 0$

$\Rightarrow b(4b-2) = 0,\quad b = 0,\ b = 1/2$

⑤ $(v+w)\cdot(v+w) - (2v)\cdot w = v\cdot v + \cancel{v\cdot w} + \cancel{w\cdot v} + w\cdot w \cancel{-2(v\cdot w)}$

$= \|v\|^2 + \|w\|^2$

⑥ $\|e+f\|^2 = (e+f)\cdot(e+f) = \|e\|^2 + 2\,e\cdot f + \|f\|^2$

$\Rightarrow \frac{9}{4} = 1 + 2\,e\cdot f + 1 \Rightarrow e\cdot f = 1/8$

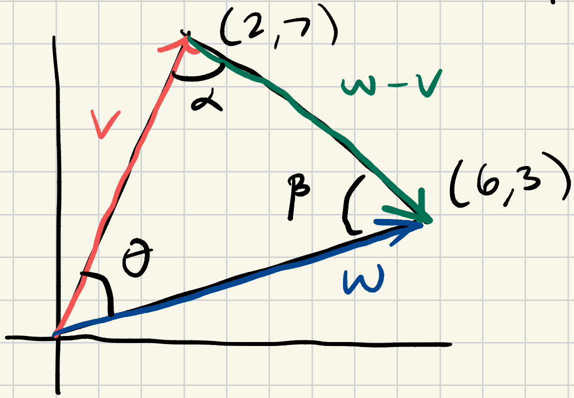
$\|e-f\|^2 = \|e\|^2 - 2\,e\cdot f + \|f\|^2 = 2 - 2\cdot\frac{1}{8} = 7/4$

$\Rightarrow \|e-f\| = \sqrt{7}/2$

⑦ $u = \langle 1,1,1\rangle, \; v = \langle 1,1,0\rangle$

$u_{\|v\|} = \frac{u\cdot v}{v\cdot v}\,v = \frac{1+1+0}{1+1}\,v = v$

⑧



θ: Angle b/w v & w

β: Angle b/w w & w−v

α: π − θ − β

$v = \langle 2,7\rangle$

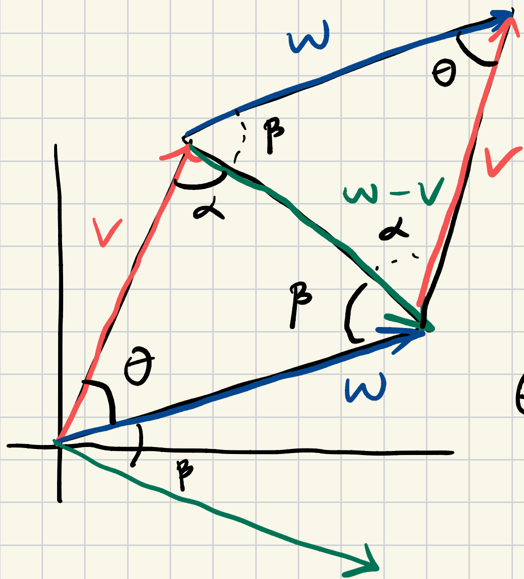
$w = \langle 6,3\rangle$

$v\cdot w = 12+21 = 33$

$\|v\| = \sqrt{4+49} = \sqrt{53}$

$\|w\| = \sqrt{36+9} = \sqrt{45}$

$\theta = \cos^{-1}\left(\frac{33}{\sqrt{53}\sqrt{45}}\right) \approx 0.829$ rad.



$w - v = \langle 4, -4 \rangle \quad \|w - v\| = \sqrt{16+16} = \sqrt{32}$

$(w - v) \cdot w = 24 - 12 = 12$

$\beta = \cos^{-1}\left(\frac{12}{\sqrt{32}\sqrt{45}}\right) \approx 1.25 \text{ rad}$

$\alpha = \pi - \theta - \beta \approx 1.063 \text{ rad}$

⑨ If $\|v + w\| = \|v - w\|$, then $\|v + w\|^2 = \|v - w\|^2$

$\Rightarrow (v+w)\cdot(v+w) = (v-w)\cdot(v-w)$

$\Rightarrow v \cdot v + v \cdot w + w \cdot v + w \cdot w = v \cdot v - v \cdot w - w \cdot v + w \cdot w$

$\Rightarrow \cancel{\|v\|^2} + 2 v \cdot w + \cancel{\|w\|^2} = \cancel{\|v\|^2} - 2 v \cdot w - \cancel{\|w\|^2}$

$\Rightarrow 2 v \cdot w = -2 v \cdot w \Rightarrow 4 v \cdot w = 0 \Rightarrow v \cdot w = 0$

So v is orthogonal to w